Comment on "Strongly correlated Fractional Quantum Hall line junction"

Letter [1] and subsequent extended paper proposed an "exact solution" of the problem of a tunnel junction of length L between two single-mode edges of the Fractional Quantum Hall Liquids with different filling factors $\nu_j = 1/(2m_j+1)$, $j=1,2$; $m_j = 0,1,\dots$. The authors use a special choice of parameters which makes possible [2, 3] simple description of the tunneling part of such a "line" junction using fermionization, and argue that this choice is justified in the limit of infinitely strong but local Coulomb interaction of the closely-spaced edges. The purpose of this comment is to point out that the solution found in [1] is not correct and to present correct solution of a particular instance of this model. One important consequence of our solution is that the maximum tunneling conductance G of the line junction has the same "strong-coupling" value $G = G_m \equiv 2\nu_1\nu_2/(\nu_1+\nu_2)$ (in units of the free electron conductance $\sigma = e^2/h$) as a point contact [4], and not the smaller value $G_J = \min\{\nu_{m_1}, \nu_{m_2}\}$ obtained in [1].

The mistake in [1] is caused by unjustified assumption of the existence of the local chemical potentials not only for the incoming but also for the outgoing edges. This assumption is important since the junction conductance G is calculated in [1] by matching these potentials across the ends of the junction. Existence of the chemical potentials implies that the local equilibrium is imposed at the junction end points $x = \pm L/2$, while actually there is no equilibrium: the chemical potentials are defined only for the incoming edges, and all the rest follows from coherent quantum evolution of the fields governed by the edge Hamiltonian [5].

Correct matching between the junction and external edges consists in imposing the continuity of the edge Bose fields $\phi_j(x,t)$ at $x=\pm L/2$ and coincides with the standard "unfolded" form [6] of multi-component Dirichlet boundary condition. The fields $\phi_{1,2}$ are normalized so that the edge currents are $j_i = -e\sqrt{\nu_i}\partial_t\phi_i/2\pi$. Under the transformation [1, 2, 3] of $\phi_{1,2}$ into the "charge" and "spin" modes $\phi_{c,n}(x,t)$ propagating independently inside the junction $|x|<L/2$, the continuity conditions in the notations of [1] have the following matrix form:

$$\phi_{c,n}(x,t) = M\phi_{1,2}(x,t)|_{x=\pm L/2}\,,$$

$$M = \frac{1}{\sqrt{\nu_1-\nu_2}}\begin{pmatrix}\sqrt{\nu_1}\,, & \sqrt{\nu_2}\\ \sqrt{\nu_2}\,, & \sqrt{\nu_1}\end{pmatrix}. \qquad (1)$$

taking $\nu_1 > \nu_2$. The charge mode $\phi_c(x - v_c t)$ is a free chiral field which moves with some velocity v_c in the same, "positive", direction as ϕ_1. Hence, the field values at the end points are related as $\phi_c(L/2, t+L/v_c) = \phi_c(-L/2,t)$. The spin mode $\phi_n(x)$ has the opposite chirality. Its dynamics is affected by tunneling and can be solved by refermionization [2, 3]. We assume that the tunnel amplitude Δ is constant throughout the junction, and limit our discussion here to the simplest case $m_2 - m_1 = 1$, when the fermionic representation of the junction dynamics [2] has the form of the tunneling-induced rotation between two components of a Dirac fermion propagating with a velocity v_n. The spin mode is defined by the difference between the density operators of the two components.

Although this fermionic representation does not provide a general simple relation between the spin mode operators at the boundaries, for special values of the junction length: $L = (\pi v_n/2\Delta)l$, where $l = 1,2\dots$, it shows that $\phi_n(-L/2, t+L/v_n) = (-1)^l\phi_n(L/2,t)$. Combined with Eq. (1), this relation gives complete description of the dynamics of the fields ϕ in the line junction. This dynamics exhibits multiple interference governed by the reflection amplitudes $\sqrt{\nu_2/\nu_1}$ at the end points resembling [7] a Tomonaga-Luttinger wire connected to leads.

To find the zero-frequency conductance G, one can ignore the finite times of propagation between the end points. This means that for even l the fields $\phi_{1,2}$ are not changed and $G=0$. For odd l, the spin mode changes sign and the total transformation T of the fields $\phi_{1,2}$ is:

$$\phi_{1,2}|_{x=L/2} = T\phi_{1,2}|_{x=-L/2}\,, \quad T = M^{-1}\sigma_z M\,. \qquad (2)$$

Using Eq. (1), and changing the transfer matrix T (2) into the scattering matrix P which relates incoming and outgoing fields ϕ, one can immediately see that P coincides with the scattering matrix of a point contact [4] (or in fact any odd number of successive point contacts [8]) in the strong-coupling limit. In this case the junction conductance is $G = G_m$ contrary to the result obtained in [1] (see Eqs. (14) and (40)-(42) of, respectively, the short and long papers). Qualitatively, based on the nature of the fermionic dynamics inside the junction, one can expect G to oscillate with L between the maximum G_m and the minimum $G=0$.

Vadim V. Ponomarenko
International Center for Condensed Matter Physics, Universidade de Braslia, 70910-900 Braslia, Brazil, and Center for Advanced Studies, St. Petersburg State Polytechnical University, St. Petersburg 195251, Russia.
Dmitri V. Averin
Department of Physics and Astronomy
Stony Brook University, SUNY
Stony Brook, NY 11794-3800, USA

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